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Quantum-jump approach to dipole dephasing: Application to inversionless amplification

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Abstract. We present a model for simulating atomic dipole dephasing within the Monte-Carlo wave function formalism which leads to the same results as the standard density-matrix formalism. In this model dipole dephasing processes are accounted by quantum-jumps into one of the atomic energy eigenstates. This fact allows us to obtain analytical expressions for the effects of elastic collisions on the various physical processes responsible for amplification without population inversion in a cascade three-level atomic configuration.

PACS. 42.50.-p Quantum optics – 32.80.-t Photon interactions with atoms – 02.70.Lq Monte-Carlo and statistical methods

1 Introduction

In general, it is not possible to describe in terms of a simple atomic wave-function the dynamics of a system consisting of an atom interacting with a coherent electromagnetic field when dissipative processes due to coupling with a reservoir are included. Alternatively, one can consider the evolution of the reduced atomic density-matrix, thus allowing to solve the problem for an averaged ensemble. The reduced density-matrix gives a quantitative description of the behavior of the system but does not provide, in general, a clear physical interpretation of the mechanisms involved in the light-matter interaction. Recently, a Monte-Carlo Wave Function (MCWF) formalism [1–4] has been proposed as a model to picture the dynamics of an atom submitted to coherent laser fields and to dissipative processes. In this formalism the time evolution of the wave-function of a single atom, a so-called quantum trajectory, is calculated by integrating the time-dependent Schrödinger equation using an effective non-Hermitian Hamiltonian. Incoherent processes are incorporated as quantum-jumps or wave-function collapses occurring at random times. Thus, a quantum trajectory consists of a series of coherent evolution periods separated by quantum-jumps occurring at random times. The MCWF formalism, which is equivalent to the density-matrix formalism when averaged over many realizations of the trajectories, is interesting at least for two different reasons: (i) in the wave-function treatment of a system belonging to a N dimensional Hilbert space the number of variables is $\sim N$ while in the density-matrix is $\sim N^2$ and (ii) it provides new insights into the underlying physical mechanisms. The MCWF formalism has been applied successfully to a large number of problems in quantum optics ranging from laser cooling [5] and the micromaser [6] to amplification without inversion [7–10] in connection with the quantum Zeno effect [11]. Also, in some limited cases, to describe systems coupled to non-Markovian reservoirs [12].

Based on the MCWF formalism, Cohen-Tannoudji *et al.* [7] derived general statistical properties of the coherent evolution periods occurring between two successive quantum-jumps. For this analysis to be applicable two conditions are required: (i) the number of relevant atomic states involved in the dissipative processes has to be finite, and (ii) the Hamiltonian has to be time independent. Using this procedure, Cohen-Tannoudji *et al.* [7] obtained analytically, *i.e.* without requiring explicitly a Monte-Carlo simulation, the respective contributions of the various physical mechanisms responsible for inversionless amplification of a probe field in a driven Λ-type three-level system [13] in the presence of incoherent pump processes and spontaneous emission.

Another important cause of dissipation is dipole dephasing due to, *e.g.*, elastic collisions. Nevertheless, the analytical method developed in [7] cannot be applied to study the influence of atomic dipole dephasing, if these processes are introduced in the standard way described in [4]. The reason is that in [4], the quantum-jumps associated with dipole dephasing collapse the atomic wave-function, that is in general a linear superposition of atomic energy eigenstates, into another superposition state. Therefore, the number of atomic states involved in these dissipative processes is infinite, even for a finite number of atomic energy eigenstates. In this paper we present an alternative method to introduce dipole dephasing

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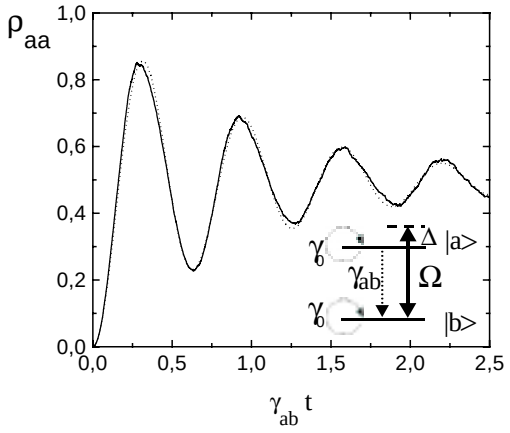

Fig. 1. Damped Rabi oscillations for the two-level system displayed in the inset for an average of 500 MCWF simulations (solid line) and corresponding density-matrix solution (dotted line). The parameters are: $\Omega = 5\gamma_{ab}$, $\gamma_0 = 0.5\gamma_{ab}$ and $\Delta = 0$.

processes in the MCWF formalism in such a way that these processes collapse the atomic wave-function into one of the atomic energy eigenstates and, therefore, the analysis of reference [7] can be applied in a straightforward way to obtain the effects of the dipole dephasing processes on the different physical processes responsible for amplification or absorption of light by atoms.

In Section 2 we present our alternative method to simulate atomic dipole dephasing in the MCWF formalism and demonstrate its equivalence with the density-matrix formalism. In Section 3, we apply the quantum-jump method developed in reference [7] to study the role of elastic collisions in inversionless amplification in a cascade three-level system. Finally, we summarize the main results of this paper in the conclusions.

2 Model

Let us consider a two-level system (inset Fig. 1) being $|a\rangle$ ($|b\rangle$) the excited (ground) state and $|\phi\rangle = c_a |a\rangle + c_b |b\rangle$ the wave-function of the atomic system. In reference [4] the quantum-jumps associated with dipole dephasing correspond to change the sign of c_b and to leave the sign of c_a unchanged. This MCWF procedure to simulate dipole dephasing is easy to implement numerically and, when averaged over a large ensemble of simulations, gives the same results as the density-matrix formalism. Nevertheless, it has the inconvenience that quantum-jumps associated to dipole dephasing collapse the wave-function to an infinite number of different final states. As previously stated, this fact prevents to study analytically the stochastic sequence of coherent evolution periods taking place between quantum-jumps. In addition, it is not possible to associate these dipole dephasing processes to exchange of quanta between the two-level atom and the electromagnetic field since the state appearing when a dephasing process collapses the wave-function is not an energy eigenstate.

Consider now a multilevel atomic system driven by coherent laser fields. The atomic wave-function at time t is $|\phi(t)\rangle = \sum_n c_n(t) |n\rangle$, $|n\rangle$ being the eigenstates of the atomic Hamiltonian. Dipole dephasing processes do not change the internal energy of the system. For this reason, we propose to model dephasing processes as random collapses of the wave-function in any of the states $|n\rangle$ with a probability proportional to a common dephasing rate, γ_0, times the population of the corresponding state $|n\rangle$. As usual in the MCWF formalism, the Hamiltonian of the system has to be modified by considering an effective non-hermitian Hamiltonian: $H_{eff} = H_0 - (i/2)\gamma_0 \sum_n |n\rangle \langle n|$, where H_0 is the Hamiltonian describing, in the absence of dissipative processes and in the rotating wave approximation, the evolution of the coherently driven atomic system ($\hbar = 1$).

To be more specific, let us consider a two-level system (see inset to Fig. 1) in interaction with a coherent field with Rabi frequency 2Ω and detuning Δ, and subjected to two different sources of dissipative processes: spontaneous emission with a relaxation rate γ_{ab} and dipole dephasing, *e.g.* elastic collisions, with a rate γ_0. At time t, the normalized wave-function of the two-level system is $|\phi(t)\rangle = c_a(t) |a\rangle + c_b(t) |b\rangle$. At $t + dt$, the wave-function evolves towards one of these states:

$$|\phi(t+dt)\rangle = \begin{cases} \left|\phi^{(0)}(t+dt)\right\rangle \\ \left|\phi^{(1a)}(t+dt)\right\rangle \\ \left|\phi^{(1b)}(t+dt)\right\rangle \\ \left|\phi^{(1c)}(t+dt)\right\rangle \end{cases}. \tag{1}$$

The first state $\left|\phi^{(0)}(t+dt)\right\rangle$ accounts for a continuous coherent evolution, *i.e.* time evolution without wave-function collapse. This coherent evolution corresponds to the non-unitary evolution given by $\left|\phi^{(0)}(t+dt)\right\rangle = (1 - idtH^{(1)}_{eff}) |\phi(t)\rangle$ with the following effective non-Hermitian Hamiltonian

$$H^{(1)}_{eff} = H_0 - (i/2)(\gamma_{ab} + \gamma_0)\sigma^+\sigma^- - (i/2)\gamma_0\sigma^-\sigma^+,$$

being $\sigma^+ = |a\rangle \langle b|$ and $\sigma^- = |b\rangle \langle a|$ the atomic raising and lowering operators, respectively. The other three states in (1) account for the wave-function collapses or quantum-jumps. There are three different possible quantum-jumps associated with: (i) spontaneous emission from $|a\rangle$ to $|b\rangle$ (ii) dipole dephasing collapsing the wave-function in $|b\rangle$ proportionally to the population of state $|b\rangle$, and (iii) dipole dephasing collapsing the wave-function in $|a\rangle$ proportionally to the population of state $|a\rangle$. Thus, $\left|\phi^{(1a)}(t+dt)\right\rangle \propto |b\rangle$ corresponds to the atomic wave-function collapse in the ground state associated with the spontaneous emitted photon and its norm is given by

$$\left\langle \phi^{(1a)}(t+dt) | \phi^{(1a)}(t+dt) \right\rangle = dp_a = \gamma_{ab} c_a c_a^* dt = \gamma_{ab} dt \langle \phi(t)| \sigma^+\sigma^- |\phi(t)\rangle .$$

For the quantum-jumps associated with dipole dephasing we have $\left|\phi^{(1b)}(t+dt)\right\rangle \propto |b\rangle$ with

$$\left\langle \phi^{(1b)}(t+dt)|\phi^{(1b)}(t+dt)\right\rangle = dp_b = \gamma_0 c_b c_b^* dt = \gamma_0 dt \left\langle \phi(t)\right| \sigma^- \sigma^+ \left|\phi(t)\right\rangle ,$$

and $\left|\phi^{(1c)}(t+dt)\right\rangle \propto |a\rangle$ with

$$\left\langle \phi^{(1c)}(t+dt)|\phi^{(1c)}(t+dt)\right\rangle = dp_c = \gamma_0 c_a c_a^* dt = \gamma_0 dt \left\langle \phi(t)\right| \sigma^+ \sigma^- \left|\phi(t)\right\rangle .$$

Therefore, the total probability of a quantum-jump in the time interval dt is $dp = dp_a + dp_b + dp_c$ and, consequently, $\left\langle \phi^{(0)}(t+dt)|\phi^{(0)}(t+dt)\right\rangle = 1 - dp$.

As usual in the MCWF procedure, a pseudorandom number ϵ is used to propagate the wave-function from an initial time t to $t+dt$. Consider the normalized two-level atomic wave-function $|\phi(t)\rangle$ and a time interval small enough such that the probability amplitudes nearly have not evolved, *i.e.* $dt \ll \Omega^{-1}$, and for which there has been at most one-quantum-jump, *i.e.* $dt \ll \gamma_{ab}^{-1}, \gamma_0^{-1}$. For $\epsilon > dp$ the wave-function at $t+dt$ is the normalized wave-function $\left|\phi^{(0)}(t+dt)\right\rangle$, *i.e.* $|\phi(t+dt)\rangle = \mu(1 - idtH_{eff}^{(1)})\,|\phi(t)\rangle$ with $\mu = 1/\sqrt{(1-dp)}$. For $\epsilon < dp$ we use another pseudorandom number to collapse the wave-function in the ground state $|b\rangle$ or the excited state $|a\rangle$ with respective probabilities $(dp_a + dp_b)/dp$ and dp_c/dp. We will demonstrate now that this MCWF approach, when averaged over a large number of simulations, is equivalent to the standard density-matrix formalism. Let us consider the operator $\rho(t) \equiv |\phi(t)\rangle \langle\phi(t)|$. The average of $\rho(t+dt)$ over different outcomes of MCWF simulations reads:

$$\overline{\rho(t+dt)} = (1-dp)\mu^2 \left(1 - idtH_{eff}^{(1)}\right) |\phi(t)\rangle \langle\phi(t)| \times \left[1 + idt\left(H_{eff}^{(1)}\right)^\dagger\right] + (dp_a + dp_b)\,|b\rangle\langle b| + dp_c\,|a\rangle\langle a| . \quad (2)$$

Substituting the explicit value of dp_i $(i = a, b, c)$ in the above equation and neglecting terms proportional to $(dt)^2$, one obtains:

$$\overline{\rho(t+dt)} \simeq \overline{\rho(t)} - idt\left[H_{eff}^{(1)}\overline{\rho(t)} - \overline{\rho(t)}\left(H_{eff}^{(1)}\right)^\dagger\right] + \gamma_{ab} dt \sigma^- \overline{\rho(t)} \sigma^+ + \gamma_0 dt(\sigma^-\sigma^+\overline{\rho(t)}\sigma^-\sigma^+ + \sigma^+\sigma^-\overline{\rho(t)}\sigma^+\sigma^-) \quad (3)$$

where $\overline{\rho(t)}$ is the average of $|\phi(t)\rangle\langle\phi(t)|$ over different MCWF simulations. Therefore, equation (3) gives:

$$\frac{d\overline{\rho}}{dt} = -i\left[H_0, \overline{\rho}\right] + \hat{A} + \hat{B} \quad (4)$$

with:

$$\hat{A} = -\frac{\gamma_{ab}}{2}\left(\overline{\rho}\sigma^+\sigma^- + \sigma^+\sigma^-\overline{\rho} - 2\sigma^-\overline{\rho}\sigma^+\right), \quad (5a)$$

$$\hat{B} = -\frac{\gamma_0}{2}\left[\overline{\rho}\left(\sigma^+\sigma^- + \sigma^-\sigma^+\right) + \left(\sigma^+\sigma^- + \sigma^-\sigma^+\right)\overline{\rho} - 2\left(\sigma^-\sigma^+\overline{\rho}\sigma^-\sigma^+ + \sigma^+\sigma^-\overline{\rho}\sigma^+\sigma^-\right)\right]. \quad (5b)$$

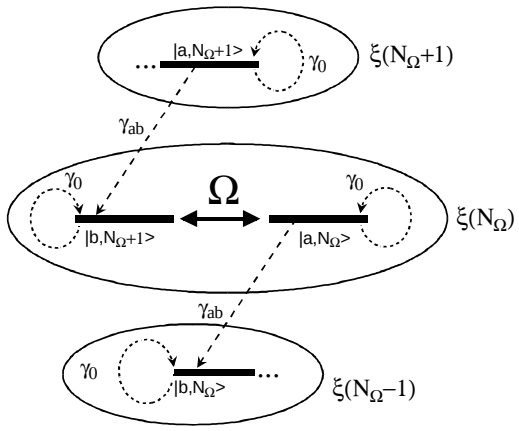

Fig. 2. Manifolds of the two-states of the atom plus laser field corresponding to the two-level system shown in the inset of Figure 1.

Taking the matrix elements of operators $\hat{A}$ and $\hat{B}$ between states $|a\rangle$ and $|b\rangle$, one obtains

$$\langle a|\hat{A}|a\rangle = -\gamma_{ab}\langle a|\overline{\rho}|a\rangle \quad \langle a|\hat{B}|a\rangle = 0 \quad (6a)$$

$$\langle b|\hat{A}|b\rangle = \gamma_{ab}\langle a|\overline{\rho}|a\rangle \quad \langle b|\hat{B}|b\rangle = 0 \quad (6b)$$

$$\langle a|\hat{A}|b\rangle = -\frac{\gamma_{ab}}{2}\langle a|\overline{\rho}|b\rangle \quad \langle a|\hat{B}|b\rangle = -\gamma_0\langle a|\overline{\rho}|b\rangle \quad (6c)$$

$$\langle b|\hat{A}|a\rangle = -\frac{\gamma_{ab}}{2}\langle b|\overline{\rho}|a\rangle \quad \langle b|\hat{B}|a\rangle = -\gamma_0\langle b|\overline{\rho}|a\rangle . \quad (6d)$$

Clearly while operator $\hat{A}$ accounts for the spontaneous emission, the role of operator $\hat{B}$ is to modify only the relaxation rate of the off-diagonal elements as expected for dipole dephasing processes [14]. Thus, equation (4) with expressions (5a, 5b) is the standard density-matrix equation for a coherently driven two-level system with spontaneous emission and dipole dephasing.

In order to check the previous results, Figure 1 shows the transient behavior of the excited state population for an average of 500 MCWF simulations starting in the ground state (solid line) and the corresponding density-matrix solution (dotted line) when spontaneous emission and dipole dephasing are included. One clearly sees the complete agreement between both formalisms for an ensemble of atoms.

Since the number of relevant atomic states is finite, the formalism developed in reference [7] allows to investigate analytically the exchange of quanta between the two-level atom and the driving electromagnetic field when dissipative processes are taken into account. Let us consider a quantum description of the laser field interacting with the two-level system. Figure 2 shows the states of the total system (*i.e.*, atom plus laser field) grouped into different manifolds ξ of two quasi-degenerate states. The dynamics of the system is described by a series of coherent evolution periods (represented by solid horizontal arrows) interrupted at random times by quantum-jumps corresponding to dissipative processes (represented by dashed

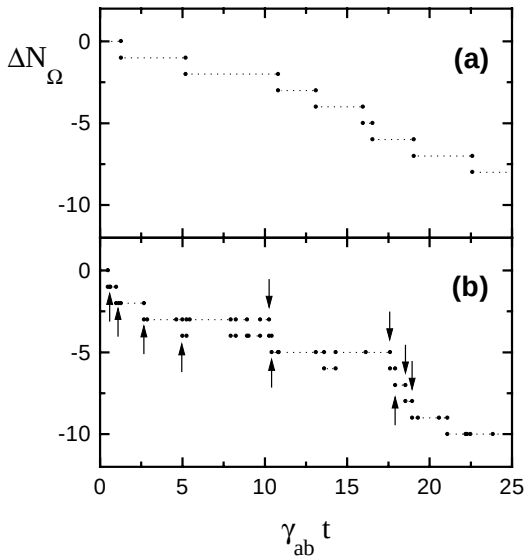

Fig. 3. Stochastic evolution of the number of photons of the coherent field for $\Omega = 3\gamma_{ab}$, $\Delta = 0$ and $\gamma_0 = 0$ (a) or $\gamma_0 = \gamma_{ab}$ (b). The solid arrows in (b) mark the time of spontaneous emission processes. The rest of the quantum-jumps are associated with dipole dephasing.

and dotted arrows). The coherent evolution periods are called period (i, j) with i and j $(= a, b)$ denoting the atomic initial and final states. The initial and the final states of these coherent evolution periods determine the energy exchange between the atom and the driving field. Thus, a one-photon loss process corresponds to a coherent evolution period starting in the atomic ground state immediately after a quantum-jump, and ending its coherent evolution in the atomic excited state, *i.e.* period (b, a). The subsequent quantum-jump can correspond to a spontaneous emission process (dashed arrow) or to a dipole dephasing process (dotted arrow). In the first case, the next coherent evolution starts in the atomic ground state while in the second case it starts in the atomic excited state. On the other hand, a one-photon gain process corresponds to a coherent evolution period starting in the atomic excited state and ending its coherent evolution in the atomic ground state by a quantum-jump associated to a dipole dephasing process, *i.e.* period (a, b). Clearly, in the absence of dipole dephasing processes (dotted arrows), there are only one-photon loss processes since all coherent evolution periods start in the atomic ground state and end in the atomic excited state.

Figure 3 shows a MCWF simulation for the two-level system of Figure 1 (inset) for two different rates of dipole dephasing processes. We plot the variation in the number of driving photons only at the time of the quantum-jumps (circles), when it has a well-defined value. The dotted line between quantum-jumps is plotted in order to guide the eyes but lacks of any physical meaning. Indeed, during the coherent evolution periods the system is in a superposition of the two states of the corresponding manifold and, therefore, the number of driving photons is not well-defined. One can notice in Figure 3 that the number of driving photons is decreasing in the two cases shown, as expected for a two-level medium without population inversion. However, while for $\gamma_0 = 0$ (Fig. 3a) there are only one-photon loss processes, for $\gamma_0 \neq 0$ (Fig. 3b) there are also one-photon gain processes. In Figure 3b the quantum-jumps describing spontaneous emission are marked with a solid arrow. The rest of quantum-jumps are associated with dipole dephasing. We notice that the main role of dipole dephasing processes is to increase the fluctuations in the driving field photon number. On the other hand, it is well-known that the laser linewidth effects can be incorporated by introducing an extra dipole dephasing rate if the laser fluctuations are due to phase-diffusion [15]. Our approach suggests that laser linewidth effects associated with fluctuations of the laser intensity could also be described by the same procedure.

3 Inversionless amplification in three-level systems

We now discuss along the lines developed in reference [7] the effect of elastic collisions in amplification without population inversion (AWI) in the three-level cascade scheme of Figure 4 (inset). The lower transition is driven by an intense coherent field with Rabi frequency 2β and detuning Δ_β from atomic resonance, while the upper transition is probed by a weak coherent field with Rabi frequency 2α and detuning Δ_α. We consider the following sources of dissipation: spontaneous emission in the probed and driven transitions with relaxation rates γ_{12} and γ_{23}, respectively; incoherent pumping in the probed transition with a two-way population transfer rate Λ; and elastic collisions at a rate γ_0. By using the standard density-matrix formalism, one can show for this configuration that for any set of parameter values, neither the probed nor the driven transition are inverted in the steady-state [10]. This means that in the steady state the upper state $|1\rangle$ has the smallest population, therefore, there is no population inversion at the one-photon probe transition $|1\rangle \leftrightarrow |2\rangle$ nor at the two-photon transition $|1\rangle \leftrightarrow |3\rangle$. This particular scheme in the absence of dipole dephasing processes was studied in [10] showing that, under appropriate driving field detuning, a weak incoherent pump rate Λ is enough to achieve on-resonance AWI if γ_{23} is larger than γ_{12} [16]. We will study now to which extent this result holds in the presence of elastic collisions. As shown in Figure 4, we make use of a quantum description of the two laser fields by considering the number of photons of driving, N_β, and probe, N_α, fields. The states of the total system atom plus laser fields are grouped into different manifolds of three states. The dynamics of the system is pictured in the MCWF formalism as consisting of a series of coherent evolution periods separated by quantum-jumps occurring at random times. The coherent evolution periods are called period (i, j) with i and j $(= 1, 2, 3)$ denoting the atomic initial and final states, respectively. As a general feature, spontaneous emission and incoherent pumping (dashed oblique lines) correspond to quantum-jumps

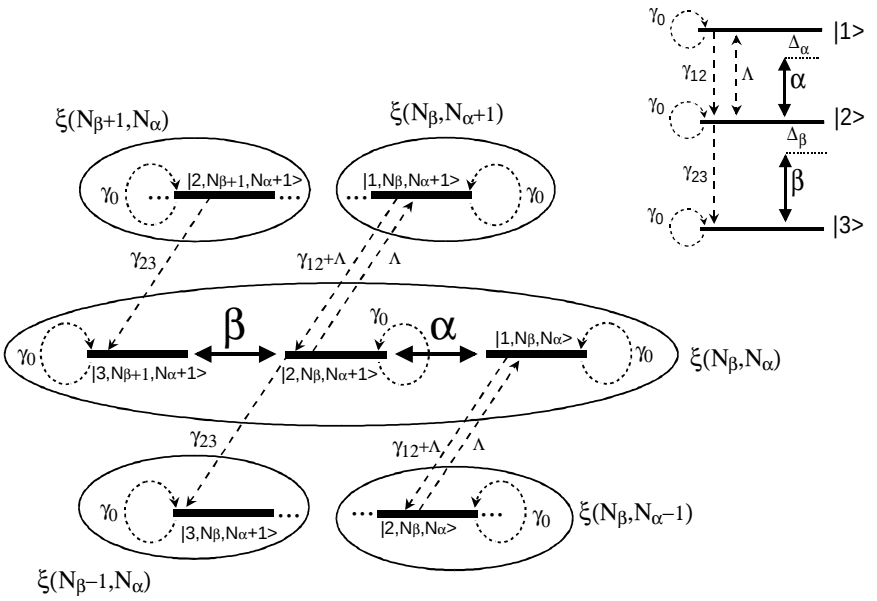

Fig. 4. Manifolds of the three states of the atom plus laser photon system corresponding to the cascade scheme displayed in the inset.

connecting different manifolds while elastic collisions (dotted circular lines) yield a new coherent evolution period in the same manifold as the previous one. There are four different periods that change the number of photons of the probe field:

$$\text{period}(1,2) \longrightarrow \Delta N_\beta = 0, \quad \Delta N_\alpha = 1, \tag{7a}$$

$$\text{period}(2,1) \longrightarrow \Delta N_\beta = 0, \quad \Delta N_\alpha = -1, \tag{7b}$$

$$\text{period}(1,3) \longrightarrow \Delta N_\beta = 1, \quad \Delta N_\alpha = 1, \tag{7c}$$

$$\text{period}(3,1) \longrightarrow \Delta N_\beta = -1, \quad \Delta N_\alpha = -1. \tag{7d}$$

Periods $(1,2)$ and $(2,1)$ correspond, respectively, to one-photon stimulated gain and loss processes, while periods $(1,3)$ and $(3,1)$ account, respectively, for two-photon stimulated gain and loss processes. Therefore, the mean change of the probe field photon number per period can be split in one-photon and two-photon contributions, $\langle \Delta N_\alpha \rangle = \langle \Delta N_\alpha \rangle_{1p} + \langle \Delta N_\alpha \rangle_{2p}$, with $\langle \Delta N_\alpha \rangle_{1p} = P(1,2) - P(2,1)$ and $\langle \Delta N_\alpha \rangle_{2p} = P(1,3) - P(3,1)$, where $P(i,j)$ is the probability that a coherent evolution period randomly selected from a quantum trajectory starts in $|i\rangle$ and ends in $|j\rangle$. As shown in [7], the probabilities $P(i,j)$ read:

$$P(i,j) = P(i) G_j \int_0^\infty |c_{ij}|^2 \, d\tau \tag{8}$$

where $P(i)$ is the probability that a coherent evolution period starts in state $|i\rangle$, G_j is the total departure rate from state $|j\rangle$ through a quantum-jump, *i.e.* $G_1 = \gamma_{12} + \Lambda + \gamma_0$, $G_2 = \gamma_{23} + \Lambda + \gamma_0$ and $G_3 = \gamma_0$, and, finally, c_{ij} is the probability amplitude to find the atom in state $|j\rangle$ at time $t + \tau$ when it started its coherent evolution in state $|i\rangle$ at time t, *i.e.* $c_{ij}(\tau) = \langle j | \exp(-iH_{eff}\tau) | i \rangle$. For the cascade scheme under investigation, the effective non-Hermitian Hamiltonian reads:

$$\begin{aligned} H_{eff}^{(2)} = {} & (-iG_1/2 + \Delta_\alpha + \Delta_\beta) |1\rangle\langle 1| \\ & + (-iG_2/2 + \Delta_\beta) |2\rangle\langle 2| - iG_3/2 \, |3\rangle\langle 3| \\ & + \alpha (|1\rangle\langle 2| + |2\rangle\langle 1|) + \beta (|2\rangle\langle 3| + |3\rangle\langle 2|) . \end{aligned} \tag{9}$$

The probability amplitudes c_{ij} satisfy the symmetric property $c_{ij} = c_{ji}$ since $\left(H_{eff}^{(2)}\right)^\dagger = \left(H_{eff}^{(2)}\right)^*$ [7]. In the steady-state regime, the probabilities $P(i)$ can be obtained through the recursive relation: $P(i) = \sum_j P(j) Q(in : i/in : j)$ being $Q(in : i/in : j)$ the conditional probability to start a coherent evolution period in state $|i\rangle$ once the previous one has started in state $|j\rangle$. In our case, taking $\alpha \ll \Lambda$ and $\beta \gg \gamma_0, \gamma_{12}, \Lambda$, one obtains after some algebraic manipulation (see Appendix):

$$\langle \Delta N_\alpha \rangle_{1p} = A \left[\Lambda (\gamma_{23} - \gamma_{12}) - \gamma_0 \gamma_{12} \right] \int_0^\infty |c_{12}(\tau)|^2 \, d\tau, \tag{10a}$$

$$\langle \Delta N_\alpha \rangle_{2p} = -A \left[\gamma_{23} (\Lambda + \gamma_{12}) + \gamma_0 \gamma_{12} \right] \int_0^\infty |c_{13}(\tau)|^2 \, d\tau, \tag{10b}$$

with $A = (\gamma_{12} + \Lambda + \gamma_0) / [(\gamma_{12} + \Lambda)(\gamma_{23} + 2\Lambda + 2\gamma_0) + \Lambda\gamma_0]$. Clearly, while two-photon processes always contribute to probe laser absorption, one-photon processes can contribute to amplification depending on the parameter values. It is important to remark that for all parameter values, population in state $|1\rangle$ is smaller than population in state $|2\rangle$ which shows that there is an asymmetry between one-photon processes [10]. The physical origin of this asymmetry has been discussed in detail in reference [8]. In order to decrease the total number of two-photon processes, one should tune the driving field relatively far from $|2\rangle$–$|3\rangle$ resonance but maintaining the probe field on-resonance. In this way, the two-photon resonance condition is not fulfilled, which means $\int_0^\infty |c_{13}(\tau)|^2 \, d\tau \ll \int_0^\infty |c_{12}(\tau)|^2 \, d\tau$, and, consequently, the required condition for AWI reads: $\gamma_{23} > \gamma_{12}$ and $\Lambda > \gamma_0\gamma_{12} / (\gamma_{23} - \gamma_{12})$. Therefore, the main role of elastic collisions is to establish an incoherent pump threshold below which resonant AWI is not possible.

4 Conclusions

In conclusion, we have presented a method to introduce atomic dipole dephasing in the MCWF formalism and demonstrated its equivalence to the standard density-matrix formalism. In our method, the collapse of the wave-function due to dipole dephasing can only lead to a finite number of states. Because a finite number of states is involved in the quantum-jumps associated with dipole dephasing, one can use the quantum-jump technique developed in reference [7]. We have applied this procedure to discuss inversionless amplification in a cascade three-level system in presence of elastic collisions showing that their main role is to establish an incoherent pump threshold below which amplification without population inversion is not possible for a resonant probe field. Our analysis allowed us to calculate the relative contributions of the various physical processes which are involved in inversionless probe amplification, two-photon gain, two-photon loss, induced emission and absorption. Many other interesting problems in quantum-optics such as laser cooling or the micromaser with dephasing processes accounting for elastic collisions between the active atoms, or with a buffer gas or even with the walls of the maser cavity could be studied in a similar way.

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Appendix

In the limit $\alpha \ll \Lambda$ and $\beta \gg \gamma_0, \gamma_{12}, \Lambda, \Delta_\beta$ it is easy to see from Figure 4 that the conditional probability to start a coherent evolution period in state $|i\rangle$ once the previous one has started in state $|j\rangle$, denoted by $Q(in : i/in : j)$, read:

$$
\begin{aligned}
Q(in:1/in:1) &= \gamma_0/\left(\gamma_0+\gamma_{12}+\Lambda\right)\\
Q(in:2/in:1) &= \left(\gamma_{12}+\Lambda\right)/\left(\gamma_0+\gamma_{12}+\Lambda\right)\\
Q(in:3/in:1) &= 0\\
Q(in:1/in:2) &= Q(in:1/in:3) = \Lambda/\left(2\gamma_0+\gamma_{23}+\Lambda\right)\\
Q(in:2/in:2) &= Q(in:2/in:3) = \gamma_0/\left(2\gamma_0+\gamma_{23}+\Lambda\right)\\
Q(in:3/in:2) &= Q(in:3/in:3)\\
&= \left(\gamma_0+\gamma_{23}\right)/\left(2\gamma_0+\gamma_{23}+\Lambda\right).
\end{aligned}
\tag{A.1}
$$

In the steady-state, the conditional probability $P(i)$ to start a coherent evolution period in state i of any manifold is given by the recursive formula

$$P(i) = \sum_j P(j) Q(in : i/in : j)$$

which means:

$$
\begin{aligned}
P(1) &= \frac{\Lambda\left(\gamma_0+\gamma_{12}+\Lambda\right)}{\left(\gamma_{12}+\Lambda\right)\left(2\gamma_0+\gamma_{23}+2\Lambda\right)+\gamma_0\Lambda}\\
P(2) &= \frac{\left(\gamma_0+\Lambda\right)\left(\gamma_{12}+\Lambda\right)}{\left(\gamma_{12}+\Lambda\right)\left(2\gamma_0+\gamma_{23}+2\Lambda\right)+\gamma_0\Lambda}\\
P(3) &= \frac{\left(\gamma_0+\gamma_{23}\right)\left(\gamma_{12}+\Lambda\right)}{\left(\gamma_{12}+\Lambda\right)\left(2\gamma_0+\gamma_{23}+2\Lambda\right)+\gamma_0\Lambda}.
\end{aligned}
\tag{A.2}
$$

Substituting (A.2) into equation (8) it is straightforward to obtain the one-photon and two-photon contributions as given in equations (10a, 10b).

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